ELSEVIER

Journal of Materials Processing Technology 182 (2007) 262–267

Journal of Materials Processing Technology

www.elsevier.com/locate/jmatprotec

The optimization of the variable binder force in U-shaped forming with uncertain friction coefficient

C. Jiang [a], X. Han [a,*], G.R. Liu [b], G.Y. Li [a]

[a] State Key Laboratory of Advanced Design and Manufacture for Vehicle Body, Hunan University, Changsha City 410082, PR China

[b] Center for Advanced Computations in Engineering Science (ACES), Department of Mechanical Engineering, National University of Singapore, 10 Kent Ridge, Crescent, Singapore 119260, Singapore

Received 16 March 2006; received in revised form 31 July 2006; accepted 5 August 2006

Abstract

In this paper, an uncertain optimization method is suggested to obtain the optimal variable binder force in U-shaped forming. The friction coefficient is regarded as the uncertain coefficient, and the stepped variable binder force model is used. The finite element method is employed to simulate the forming process, and an uncertain objective function which represents the springback magnitude is created. The uncertain friction coefficient is treated as an interval, no need to know its probabilistic distribution. Through a nonlinear interval number programming method, the uncertain optimization problem is converted into a deterministic two-objective optimization problem. A hybrid optimization algorithm based on the intergeneration projection genetic algorithm and neural network is used to obtain the optimum. The presented method is applied to optimize the variable binder force parameters of the model from NUMISHEET'93. The forming quality based on the optimal variable binder force from the presented method is compared with constant binder force. The results indicate that the presented method can find the fair variable binder force to obtain both of the small springback and strain under the uncertain friction coefficient.

© 2006 Elsevier B.V. All rights reserved.

Keywords: Springback; U-shaped forming; Variable binder force; Uncertain optimization; Interval number

1. Introduction

The springback is one of the major faults in sheet metal forming and it is difficult to be controlled. Springback is a very complex mechanical phenomenon that involves the material property and the processing parameters such as friction coefficient, sheet thickness, temperature, etc. The uneven distribution of stress along the sheet thickness direction relaxes during unloading, thus producing the springback [1]. The binder force is often used to reduce the springback as the springback can be decreased with the increase of the binder force, while all other processing and material parameters are held constant. However, the increased binder force causes a subsequent increase of the maximum strain in the material, and it often makes the material cracked [1]. To solve this problem, Ayres [2], Hishida and Wagoner [3], Sunseri et al. [4] proposed the stepped variable binder force trajectory to obtain both of the small springback and strain. However, they had not given an effective approach to optimize the variable binder force parameters. Cao et al. [1] used the neural network (NN) to predict the stepped variable binder force based on the punch force trajectory. Liu et al. [5] determined the variable binder force using the principle of "intermediate restraining" and the forming limit diagram. Han et al. [6] adopted the progressive NN to inversely determine the variable binder force. All of these methods were based on the finite element method (FEM) and the friction coefficient was treated as a constant. However, in the actual forming process, the friction coefficient is very difficult to measure experimentally. Thus in the numerical simulation it actually cannot be specified as a precise value. To avoid the discussion of the friction coefficient, some people just omitted its influence in numerical simulation. Nevertheless it will lead to larger deviation as the friction coefficient is an important factor influencing the springback [7]. To obtain more reliable optimal variable binder force, the friction coefficient should be regarded as an uncertain coefficient and the corresponding uncertain optimization method should be used.

In the uncertain optimization, some coefficients cannot be given the precise values. The fuzzy [8–11] and stochastic approaches [12–14] are often used to describe and treat the uncertain coefficients. In these methods, the uncertain coeffi-

* Corresponding author. Tel.: +86 731 8823993; fax: +86 731 8821445.
E-mail address: hanxu@hnu.cn (X. Han).

0924-0136/$ – see front matter © 2006 Elsevier B.V. All rights reserved.
doi:10.1016/j.jmatprotec.2006.08.002

cients are regarded as the random variables or fuzzy sets based on the known probability distributions or membership functions. However, it is always difficult to specify the membership function or probability distribution in the uncertain environment [15]. In recent years, the interval analysis methods of uncertainty were developed for modeling uncertain coefficients of the engineering problems, in which the bounds of the uncertain coefficients are only required. Tanaka et al. [16], Ishibuchi and Tanaka [17], Rommelfanger [18] discussed the linear programming problem with interval coefficients in the objective function. Tong [19] considered the case in which the coefficients of the objective and constraint functions are all interval numbers. He obtained the possible interval of the solution by taking the maximum value range and minimum value range inequalities as constraint conditions. Liu and Da [20] proposed an interval number optimization method based on the fuzzy constraint satisfactory degree. Ma [21] used the deterministic optimization method to obtain the bounds of the nonlinear objective function, and converted it into a three-objective optimization problem. Most of these methods focused on the linear interval number programming instead of the general nonlinear interval number programming (NINP). Though reference [21] proposed a method to treat the nonlinearity of the objective function, the low optimization efficiency effected its application to the practical engineering problems.

In this paper, an uncertain optimization method is suggested to optimize the stepped variable binder force in U-shaped forming. The FEM is employed to simulate the forming process and the friction coefficient is treated as an uncertain coefficient. The uncertainty of the friction coefficient is described by an interval, which can be easily determined through the engineering experiences and the practical forming problem. An uncertain objective function is created to minimize the springback. Based on an NINP method, a deterministic two-objective optimization problem is obtained. A hybrid optimization algorithm is suggested to seek for the optimum. The presented method is applied to the computational model of NUMISHEET'93 and the optimization results demonstrate the efficiency of this method.

2. Statement of the problem

As shown in Fig. 1, the stepped variable binder force curve is different from the constant binder force (CBF). A low binder force (LBF) is first acted on the part and it is intended to facilitate the flow of the material. At one specified percentage of the total punch displacements (PPD), a high binder force (HBF) replaces

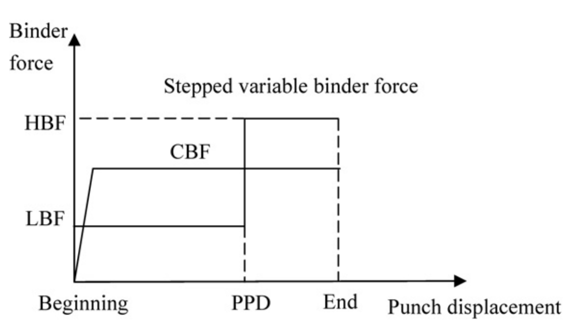

Fig. 1. The pattern of the stepped variable binder force.

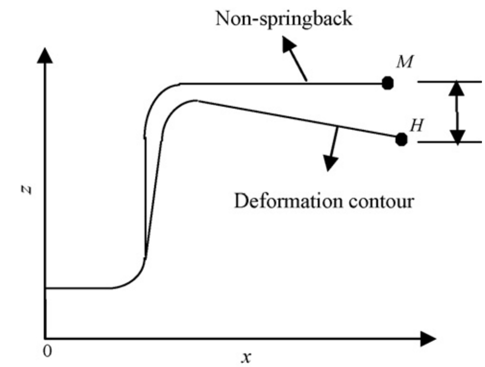

Fig. 2. The deformation contour of the U-shaped part (a half model).

the LBF and it is intended to cause plastic strains in the sidewall [6]. This stepped variable binder force curve is determined by three parameters: LBF, HBF and PPD. The LBF should be sufficient to remove wrinkling [22], while in U-shaped forming there is no wrinkling problem [5] and hence it can be specified as a small and safe value beforehand. Thus the optimization of the variable binder force is actually equivalent to the optimization of the HBF and PPD.

Fig. 2 is a half deformation contour of the part after forming. M is the bound point of the contour. H is the distance of M in z direction between the deformation contour and the non-springback contour. It is obvious that H can be used to represent the magnitude of the springback as the larger H indicates the larger springback. So the present optimization problem can be formulated to optimize the variable binder force to obtain the minimum H with uncertain friction coefficient as follows:

$$\begin{aligned} &\min f(\mathbf{X}, \mu) \\ &\mathbf{X}^T = [\text{HBL}, \text{PPD}] \in \Omega,\ \mu = [\mu^L, \mu^R] \end{aligned} \tag{1}$$

where $\mathbf{X}$ denotes the decision vector composed by HBL and PPD. Ω denotes the range of $\mathbf{X}$. μ is the uncertain friction coefficient. $[\mu^L, \mu^R]$ represents an interval number and the superscripts L, R denote the lower, upper bounds of the interval, respectively. Though the friction coefficient cannot be determined in the practical forming process, its value will not be beyond this interval. f is the objective function which represents H and it is obtained through FEM. Obviously, f is a nonlinear function of $\mathbf{X}$. For each specific $\mathbf{X}$, the possible values of f form an interval because the uncertain coefficient μ is an interval. The problem defined by Eq. (1) has been beyond the capacity of traditional optimization methods and linear interval number programming methods (e.g., [23]). In the following sections, an NINP method [24] will be introduced to solve above complex uncertain optimization problem.

3. An NINP method

The order relations are often used to compare the interval numbers in the interval number programming. They indicate that an interval number is better than another but not that one is larger than another. Ishibuchi and Tanaka [17] defined the order relation $\leq_{\text{mw}}$ between interval numbers A and B for the maximization problem:

$$\begin{aligned} &A \le_{\text{mw}} B, \quad \text{if } m(A) \le m(B) \quad \text{and} \quad w(A) \ge w(B) \\ &A <_{\text{mw}} B, \quad \text{if } A \le_{\text{mw}} B \quad \text{and} \quad A \ne B \\ &m(A) = \frac{A^L + A^R}{2}, w(A) = \frac{A^R - A^L}{2}, m(B) = \frac{B^L + B^R}{2}, w(B) = \frac{B^R - B^L}{2} \end{aligned} \tag{2}$$

where $\le_{\text{mw}}$ represents the preferences of the decision maker to the midpoint value m and the half-width w of the interval number. For the minimization problem, $\le_{\text{mw}}$ has the following form:

$$\begin{aligned} &A \le_{\text{mw}} B, \quad \text{if } m(A) \ge m(B) \quad \text{and} \quad w(A) \ge w(B) \\ &A <_{\text{mw}} B, \quad \text{if } A \le_{\text{mw}} B \quad \text{and} \quad A \ne B \\ &m(A) = \frac{A^L + A^R}{2}, w(A) = \frac{A^R - A^L}{2}, m(B) = \frac{B^L + B^R}{2}, w(B) = \frac{B^R - B^L}{2} \end{aligned} \tag{3}$$

Comparing the intervals of the objective function Eq. (1) using $\le_{\text{mw}}$ defined by Eq. (3), we expect that the optimal interval of the objective function has both of the smallest midpoint value and half-width. Therefore this uncertain objective function can be converted into a deterministic two-objective optimization problem as follows:

$$\begin{aligned} &\min[m(f(\mathbf{X}, \mu)), w(f(\mathbf{X}, \mu))] \\ &m(f(\mathbf{X}, \mu)) = \frac{1}{2}(f^L(\mathbf{X}) + f^R(\mathbf{X})) \\ &w(f(\mathbf{X}, \mu)) = \frac{1}{2}(f^R(\mathbf{X}) - f^L(\mathbf{X})) \end{aligned} \tag{4}$$

At each iterative step of $\mathbf{X}$, $f(\mathbf{X}, \mu)$ is an interval and its bounds $f^L(\mathbf{X})$, $f^R(\mathbf{X})$ can be obtained as follows:

$$\begin{aligned} &f^L(\mathbf{X}) = \min_{\mu \in \Gamma} f(\mathbf{X}, \mu), \qquad f^R(\mathbf{X}) = \max_{\mu \in \Gamma} f(\mathbf{X}, \mu) \\ &\mu \in \Gamma = \{\mu | \mu^L \le \mu \le \mu^R\} \end{aligned} \tag{5}$$

where the decision vector $\mathbf{X}$ is regarded as a constant and two deterministic optimization processes are performed with μ as the optimization variable. Through Eq. (5), the uncertain coefficient μ is removed and Eq. (4) becomes a deterministic two-objective optimization problem. The two objective functions in Eq. (4) are analogous to minimize the average value and deviation of the uncertain objective function in Eq. (1), respectively. Through minimizing the half-width, the variance of the objective function caused by the uncertain friction coefficient will be decreased. The objective function can become insensitive to the fluctuation of the uncertain coefficient. Thus it can guarantee the reliability of the optimization result.

Using the linear combination method [25], the two objective functions in Eq. (4) are formulated in terms of a desirability function $\bar{f}$ as follows:

$$\min \bar{f} = (1-\beta)(m(f(\mathbf{X}, \mu)) + \xi) + \beta(w(f(\mathbf{X}, \mu)) + \zeta) \tag{6}$$

where $0 \le \beta \le 1$ is the weight factor of the two objective functions. ξ and ζ are two parameters which make $m(f(\mathbf{X}, \mu))+\xi$ and $w(f(\mathbf{X}, \mu)) + \zeta$ non-negative. A hybrid optimization algorithm will be suggested to seek for the optimum of Eq. (6) in the following section.

4. Hybrid optimization algorithm

In this paper, the intergeneration projection genetic algorithm (IP-GA) [26] is employed as the optimization tool for Eq. (6). In the optimization process, hundreds of individuals of the decision vector $\mathbf{X}$ will be produced. For each $\mathbf{X}$, two deterministic optimization processes defined by Eq. (5) will be performed to achieve the interval of the objective function. If IP-GA is also used as the optimization tool for Eq. (5), the nesting of IP-GA will be caused and whereby the optimization efficiency will be very low. To improve the efficiency, an NN model is used to create the connections between the decision vector $\mathbf{X}$ and the interval of the objective function. Once trained, the NN model can take place the two deterministic optimization processes and output the bounds of the objective function for each $\mathbf{X}$ very quickly. The flowchart of the hybrid optimization algorithm is shown in Fig. 3.

The inputs of the NN model are $\mathbf{X}$, namely HBF and PPD. The outputs are two bounds of the objective function. The training samples for the NN model consist of a number of sets of inputs

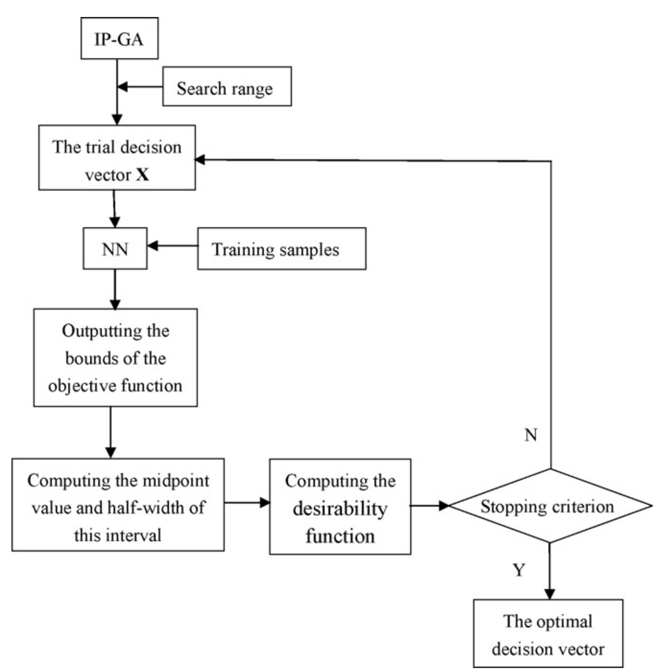

Fig. 3. The flowchart of the hybrid optimization algorithm.

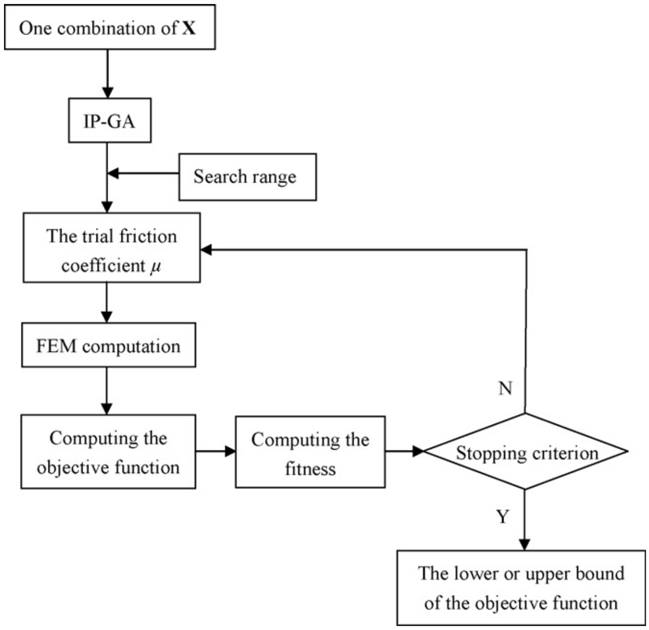

Fig. 4. The procedure to generate the training sample using IP-GA.

and outputs. All of the samples are normalized between 0.1 and 0.9 [27]. In the two-dimensional range Ω, a set of combinations of **X** are selected. For each **X**, IP-GA is used twice to obtain the bounds of the objective function based on Eq. (5). Then this **X** and the bounds of the objective function form one sample. The computational process of one sample's generation is shown in Fig. 4. It is found that **X** is fixed and the friction coefficient is used as the optimization variable in this computational process. When computing the upper bound, the value of the objective function is used as the fitness of the IP-GA; when computing the lower bound, the minus is added to the objective function and it is used as the fitness. In the hybrid optimization algorithm, constructing the samples is most time-consuming. Once the samples are obtained and the NN is trained, the NN can take place the two optimization processes defined by Eq. (5). Thus the optimization nesting can be removed and the efficiency can be improved greatly.

IP-GA combines the micro GA (μGA) [28] with the inter-generation projection (IP) operator and has a better global convergence performance [26]. An NN model is a type of computational model. It has three parts: input layer, output layer and hidden layers. In this paper, the non-linear hyperbolic functions are used as the activation functions to increase the modeling flexibility. A modified back-propagation learning algorithm with a dynamically adjusted learning rate and an additional jump factor is applied. Using this algorithm, the possible saturation of the sigmoid function can be removed and the training efficiency can be improved [29].

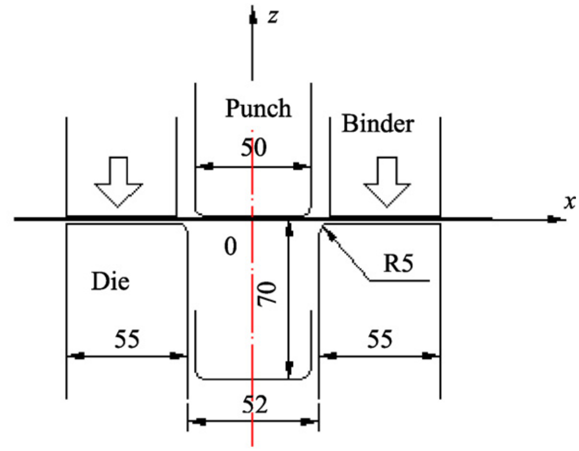

Fig. 5. The U-shaped forming of NUMISHEET'93 (unit: mm) [5].

5. The application

The presented method is applied to the computational model of NUMISHEET'93 as shown in Fig. 5. The geometry and material parameters of the model are listed in Table 1. The interval of the friction coefficient μ is specified as [0.1, 0.2]. Two-dimensional degenerated shell element [30] and elasto-plastic material model are used. The whole FEM simulation process is divided into two parts: forming and springback. The part is divided into 84 elements. As shown in Fig. 6, the elements contacting the binder are sparse, while ones contacting the tools (punch, die) are dense. A half FEM model is built because of the symmetry. The computation is accomplished by the FEM codes developed by the laboratory where the authors are working.

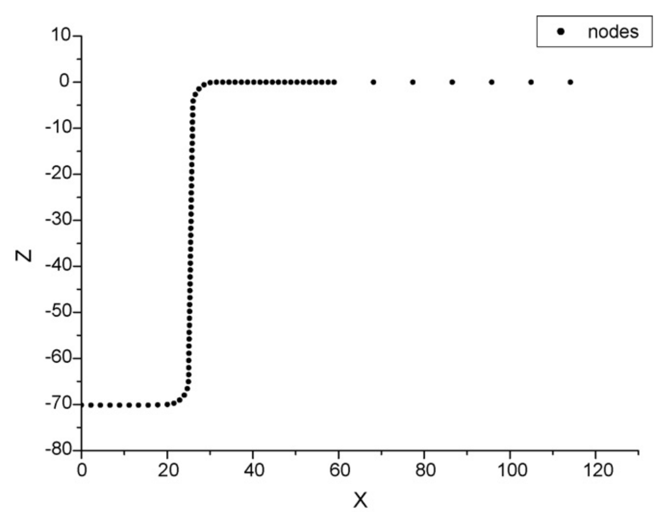

Fig. 6. The FEM mesh of the part.

Table 1
The geometry parameters and material parameters of the part

Sheet length (mm)	Sheet width (mm)	Sheet thickness (mm)	Young's modulus (GPa)	Poisson's ratio	Relation of stress and strain (MPa)
350	35	1.0	206	0.3	$\sigma = 565.2(0.007117 + \varepsilon^p)^{0.2589}$

Table 2
The comparison of the CBF and the optimal variable binder force

	Binder force (KN)		Maximum engineering major strain (%)	Maximum reduction of the thickness (%)	*H* (mm)
CBF	25.6	Lower bound	18.5	11.4	4.5
		Upper bound	22.7	13.2	7.6
Variable binder force	25.6	Lower bound	13.5	8.4	1.3
	PPD 67.5%	Upper bound	16.2	9.6	6.2

In this paper, an NN model with one hidden layer is used. The neuron numbers of the input layer, hidden layer and output layer are 2, 8 and 2, respectively. The LBF is specified as a small value 2.45 kN. A large value 40 kN is selected as the maximum of the HBF, thus the range of HBF is [2.4 kN, 40 kN]. The total punch displacement is 70 mm as shown in Fig. 5. Because the LBF is intended to facilitate the flow of the material and this stage should be kept longer, the minimum of the PPD is specified as 55%. HBF is intended to cause plastic strains in the sidewall and make the material of the flange no longer flow into the die cavity. Thus sufficient forming time should be left to HBF for plastic strains and whereby 85% is a proper value for the maximum of the PPD [6]. So the range of the PPD is specified as [55%, 85%]. The HBF and PPD are selected as 12 and 8 levels of change in the search range and hence a total of 96 training samples are used. For the IP-GA, the population size and the probability of crossover are set to 5 and 0.5, respectively. The stopping criterion is imposed to limit the IP-GA run to a maximum of 100 generations. The weight factor β is specified as 0.5. ξ and ζ are specified as 0 and 0 because $m(f(\mathbf{X}, \mu))$ and $w(f(\mathbf{X}, \mu))$ in Eq. (6) are always non-negative.

Through above mentioned uncertain optimization method, the optimum is sought as HBF = 25.6 kN, PPD = 67.5% and $\bar{f} = 3.1$ mm. In the FEM codes, the HBF and PPD are specified as 25.6 kN and 67.5%, and the friction coefficient is employed as the optimization variable. Then the upper and lower bounds of the objective function at this optimal HBF and PPD are obtained as 6.2 and 1.3 mm based on Eq. (5). In these two optimization processes, the FEM computation is called repeatedly. The corresponding friction coefficients are 0.10 and 0.16 when the objective function reaches to the upper and lower bounds, respectively. Inputting the optimal HBF and PPD as well as these two friction coefficients into the FEM codes, respectively, two deformation contours of the part are obtained as shown in Fig. 7. The contours ① and ② represent the cases with the largest and smallest springback, respectively. All possible deformation contours of the part caused by the uncertain friction coefficient at this optimal HBF and PPD will be between these two contours.

In this section the optimal variable binder force from the presented method will be compared with CBF. If the obtained optimal variable binder force can take better forming quality than traditional CBF method, we can say that the optimization result is effective. Then the presented method which generates this optimal variable binder force will also be proven effective. Here the intervals of the maximum engineering major strain str of the material and maximum reduction red of the thickness at the optimal HBF and PPD are computed through the following equations:

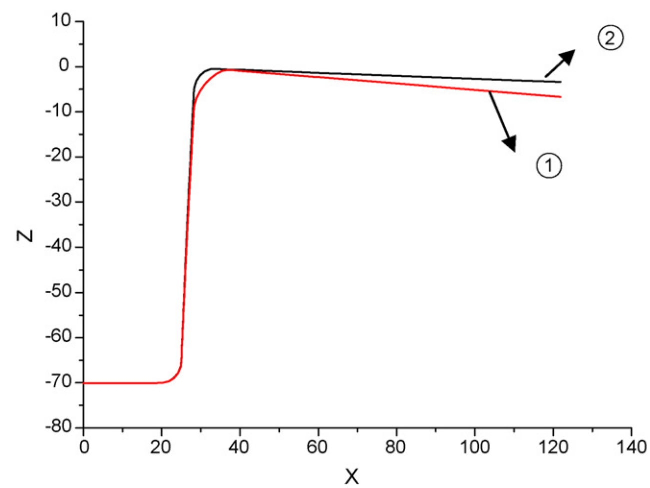

Fig. 7. The bounds of the deformation contours at the optimal variable binder force.

$$\mathrm{str}^L(\mathbf{X}_o) = \min_{\mu \in \Gamma} \mathrm{str}(\mathbf{X}_o, \mu), \quad \mathrm{str}^R(\mathbf{X}_o) = \max_{\mu \in \Gamma} \mathrm{str}(\mathbf{X}_o, \mu)$$
$$\mathrm{red}^L(\mathbf{X}_o) = \min_{\mu \in \Gamma} \mathrm{red}(\mathbf{X}_o, \mu), \quad \mathrm{red}^R(\mathbf{X}_o) = \max_{\mu \in \Gamma} \mathrm{red}(\mathbf{X}_o, \mu) \tag{7}$$

where $\mathbf{X}_o$ denotes the optimal HBF and PPD, namely HBF = 25.6 kN and PPD = 67.5%. In addition, 25.6 kN is used as the CBF to act on the sheet metal. Then through the optimization processes with the friction coefficient as the optimization variable, the bounds of the objective function, maximum engineering major strain and maximum reduction of the thickness can be also achieved. All of the results under the variable binder force and CBF are listed in Table 2. It is found that the lower bounds of the maximum engineering major strain and maximum reduction of the thickness at CBF = 25.6 kN are 18.5 and 11.4%, respectively. However, the upper bounds of these two parameters under the optimal variable binder force are only 16.2 and 9.6%, and they are both smaller than CBF's. Furthermore the interval of the objective function under the optimal variable binder force is [1.3 mm, 6.2 mm], while the one of CBF is [4.5 mm, 7.6 mm]. It indicates that using this optimal variable binder force has much less chances to be cracked. As a result, the presented method can find the good variable binder force parameters to obtain both of the smaller springback and strain with the uncertain friction coefficient.

6. Conclusion

In this paper, an uncertain optimization method is given to optimize the stepped variable binder force in U-shaped forming with uncertain friction coefficient. The friction coefficient is treated as an interval and the FEM is used to simulate the forming process. Based on the order relation of the interval number, the uncertain problem is changed into a deterministic two-objective optimization problem. A hybrid optimization algorithm is suggested to achieve the optimum. In the end, the presented method is applied to the computational model of NUMISHEET'93. The springback and strain under the optimal variable binder force and CBF are also compared. The results indicate that using the presented method a good variable binder force can be sought to obtain both of the small pringback and strain. In further study, the presented uncertain optimization method can be applied to more complex sheet metal forming problems, and more uncertain parameters can be considered.

Acknowledgements

This work is supported by the national 973 program under the grant number 2004CB719402 and the program for New Century Excellent Talents in University (NCET-04-0766).

References

[1] J. Cao, B. Kinsey, A. Solla Sara, Consistent and minimal springback using a stepped binder force trajectory and neural network control, J. Eng. Mater. Technol. Trans. ASME 122 (2000) 113–118.
[2] R.A. Ayres, SHAPESET: a process to reduce sidewall curl springback in high-strength steel rails, J. Appl. Metalworking 3 (1984) 127–134.
[3] Y. Hishida, R. Wagoner, Experimental analysis of blank holding force control in sheet forming, J. Mater. Manuf. 2 (1993) 409–415.
[4] M. Sunseri, J. Cao, A.P. Karafillis, M.C. Boyee, Accommodation of springback error in channel forming using active binder force control: numerical simulations and results, J. Eng. Mater. Technol. Trans. ASME 118 (1996) 426–435.
[5] G. Liu, Z.Q. Lin, W.L. Xu, Y.X. Bao, Variable blankholder force in U-shaped part forming for eliminating springback error, J. Mater. Process. Technol. 120 (2002) 259–264.
[6] X. Han, C. Jiang, G.Y. Li, Z.H. Zhong, An inversion procedure for determination of variable binder force in U-shaped forming, Inverse Problems Sci. Eng. 14 (2006) 301–312.
[7] L. Papeleux, J.P. Ponthot, Finite element simulation of springback in sheet metal forming, J. Mater. Process. Technol. 125–126 (2002) 785–791.
[8] M. Delgado, J.L. Verdegay, M.A. Vila, A general model for fuzzy linear programming, Fuzzy Set. Syst. 29 (1989) 21–29.
[9] M.K. Luhandjula, Fuzzy optimization: an appraisal, Fuzzy Set. Syst. 30 (1989) 257–282.
[10] R. Slowinski, A multicriteria fuzzy linear programming method for water supply systems development planning, Fuzzy Set. Syst. 19 (1986) 217–237.
[11] B.D. Liu, K. Iwamura, Fuzzy programming with fuzzy decisions and fuzzy simulation-based genetic algorithm, Fuzzy Set. Syst. 122 (2001) 253–262.
[12] A. Charnes, W.W. Cooper, Chance-constrained programming, Manage. Sci. 6 (1959) 73–79.
[13] P. Kall, Stochastic programming, Eur. J. Oper. Res. 10 (1982) 125–130.
[14] B.D. Liu, R.Q. Zhao, G. Wang, Uncertain Programming with Applications, Tsinghua University Press, Beijing, 2003.
[15] A. Sengupta, T.K. Pal, D. Chakraborty, Interpretation of inequality constraints involving interval coefficients and a solution to interval linear programming, Fuzzy Set. Syst. 119 (2001) 129–138.
[16] H. Tanaka, T. Ukuda, K. Asal, On fuzzy mathematical programming, J. Cybern. 3 (1984) 37–46.
[17] H. Ishibuchi, H. Tanaka, Multiobjective programming in optimization of the interval objective function, Eur. J. Oper. Res. 48 (1990) 219–225.
[18] H. Rommelfanger, Linear programming with fuzzy objective, Fuzzy Set. Syst. 29 (1989) 31–48.
[19] S.C. Tong, Interval number and fuzzy number linear programming, Fuzzy Set. Syst. 66 (1994) 301–306.
[20] X.W. Liu, Q.L. Da, A satisfactory solution for interval number linear programming, J. Syst. Eng. China 14 (1999) 123–128.
[21] L.H. Ma, Research on method and application of robust optimization for uncertain system, Ph. D. Thesis, Zhejiang University, China, 2002.
[22] Y.C. Liu, The effect of restraining force on shape deviation in flanged channels, J. Eng. Mater. Technol. Trans. ASME 110 (1998) 389.
[23] A. Sengupta, T.K. Pal, D. Chakraborty, Interpretation of inequality constraints involving interval coefficients and a solution to interval linear programming, Fuzzy Set. Syst. 119 (2001) 129–138.
[24] C. Jiang, X. Han, Z.H. Zhong, G.R. Liu, Nonlinear interval number programming for uncertain optimization problems, submitted for publication.
[25] Y.D. Hu, Applied Multiobjective Optimization, Shanghai Science and Technology Press, Shanghai, China, 1990.
[26] Y.G. Xu, G.R. Liu, Z.P. Wu, A novel hybrid genetic algorithm using local optimizer based on heuristic pattern move, Appl. Artif. Intell. 17 (2001) 601.
[27] G.R. Liu, X. Han, Computational Inverse Techniques in Nondestructive Evaluation, CRC Press, Florida, 2003.
[28] K. Krishnakumar, Mirco-Genetic Algorithms for Stationary and Nonstationary Function Optimization in SPIE, vol. 289, Intelligent Control and Adaptive Systems, Philadelphia, 1989.
[29] G.R. Liu, X. Han, Y.G. Xu, K.Y. Lam, Material characterization of functionally graded material by means of elastic waves and a progressive-learning neural network, Compos. Sci. Technol. 61 (10) (2001) 1401–1411.
[30] J.R. Hughes Thomas, W.K. Liu, Nonlinear finite element analysis of shells-part II. two-dimensional shells, Comp. Meth. Appl. Mech. Eng. 27 (1981) 167–181.